Handout on Logarithms and pH

ChE 1101 2005

Sometimes we run into relationships where variables show up in the power position. $4 = 2^X$ This is simple enough that we can recognize that x must equal 2. For more complicated equations, a tool was developed to solve for x – LOGARITHMIC FUNCTIONS.

(A) $x = b^y$ take the logarithm (base b) of both sides
$\log_b x = \log_b b^y$ refer to rules below to simplify
$\log_b x = y \log_b b = y$

(B) $y = \log_b x$ which is the same as $x = b^y$ but we have solved for y !!!

Another useful characteristic is that equations A and B are inverse functions meaning that the first operation "undoes" the second one and the second operation "undoes" the first one. First, let us look at a simple case you have seen many times:

$\sqrt{x^2}$ or $(\sqrt{x})^2$ & $\log_b(b^y) = y$ or $b^{\log_b y} = y$

RULES:

$\log_b 1 = 0$
$\log_b b = 1$
$\log_b a^y = y \log_b a$
$\log_{10} {}^1/_C = \log_{10} C^{-1} = -\log_{10} C$

Common logarithms are base 10 (b = 10). However, engineers frequently use natural logarithms (ln), which are base e. e is just a number equal to 2.71828……….

$\log_e x$ is denoted by ln x

So what is the inverse function?
e^x $\ln(e^x) = x$ & $e^{\ln x} = x$

Since logarithms simplify exponents, they are also useful in plotting data that spans a large range of numbers.

Time:	1	5	10	15
# yeast cells:	10	100	1000	10,000
log (#yeast cells):	1	2	3	4

pH

Now why did I say this was useful in determining pH? pH is a measure of acidity, a measure of the number of H^+ ions in the solution. But this is hard to do on a straight scale because the concentrations of H^+ ions vary by large amounts and tend to be very small numbers. In 1909, Soren Sorenson came up with a way to represent the extremely small concentrations of H^+ ions (denoted $[H^+]$) in aqueous environments. He developed the pH scale.

Remember the rule $\log_{10} {}^{1}/_{C} = \log_{10} C^{-1} = -\log_{10} C$?

Well ${}^{1}/_{C}$ is a small number just like the H^+ concentrations acids have.

$pH = -\log_{10} [H^+]$

Acidic solution: $[H^+] > 1.0 * 10^{-7}$ Molar, $pH < 7.00$
Basic solution: $[H^+] < 1.0 * 10^{-7}$ Molar, $pH > 7.00$
Neutral solution: $[H^+] = 1.0 * 10^{-7}$ Molar, $pH = 7.00$

So when $[H^+]$ decreases, pH increases.

Just a little FYI, the pH of blood in around 7.4 – does that make blood acidic or basic?

If you get the chance, read up on neat little tools and tricks used in math. It is always exciting figuring out how different things are related through math. Engineers rely on their math background to solve lots of problems and we hope you will challenge yourself in math class so that one day you can join us in the engineering profession!